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LETTER TO THE EDITORS

Comments on "Time-dependent free convection motion and heat transfer in an infinite porous medium induced by a heated sphere"

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In his recent paper, R. Ganapathy [1] obtained small-Rayleigh number solutions in the form of expansions in terms of Ra (Rayleigh number) for the problem of unsteady free convection around a sphere which is immersed in a fluid-saturated porous medium and is assumed to be suddenly heated and, subsequently, to maintain a constant heat flux at the surface. This problem, however, had already been considered by Sano and Okihara [2] using a similar approach and the present author found that the work by Ganapathy is essentially the same as that shown in [2] and, furthermore, that there are some errors in his calculation and incorrect descriptions in his discussion.

The formulation of the problem in [1] somewhat differs from that in [2]. Ganapathy expressed the solutions in the form

$$\psi/Ra = \psi_0(R,\phi,t;R_0)+Ra\psi_1(R,\phi,t;R_0)+ \quad (1)$$

$$T = T_0(R,\phi,t;R_0)+RaT_1(R,\phi,t;R_0)+ \quad (2)$$

where ϕ is the meridian angle, $\psi = \psi'/(\alpha\sqrt{K})$, $T = (T'-T_\infty)k/(Q\sqrt{K})$, $R = r'/\sqrt{K}$ and $t = \alpha t'/(K\sigma)$ are the non-dimensional stream function, temperature, radial coordinate and time, respectively, $R_0 = a/\sqrt{K}$ and $Ra = \beta g K^2 Q/(\alpha v k)$, Q, a, α, k, σ and K being the heat flux at the surface, radius of the sphere, the effective thermal diffusivity, the effective thermal conductivity, the heat capacity ratio and the permeability of the porous medium, respectively, while in [2], the solutions are expressed as

$$\psi^* = \psi_0^*(r,\phi,t^*)+Ra^*\psi_1^*(r,\phi,t^*)+ \quad (3)$$

$$T^* = T_0^*(r,\phi,t^*)+Ra^*T_1^*(r,\phi,t^*)+ \quad (4)$$

where $\psi^* = \psi/(RaR_0^3)$, $T^* = (T'-T_\infty)/(T_w-T_\infty) = T/R_0$, $r = r'/a = R/R_0$, $t^* = t/R_0$ and $Ra^* = R_0^2 Ra$. Equations (1) and (3), or (2) and (4) are different expressions of the same expansion. It is clear, however, that the expressions (3) and (4) are better than (1) and (2), since, in equations (1) and (2), two parameters Ra and R_0 appear, while in equations (3) and (4), only one parameter Ra^* appears. Moreover, the solution for ψ_0(ψ_1 in the notation used in [1]) is incorrect. The correct solution is [2]

$$\begin{aligned}\psi_0 = 2\sqrt{t}R_0^2[-(1/\sqrt{\pi})\exp[-(\eta-\eta_0)^2]+\eta_0(\eta_0\eta^{-1}-1)\\ \times\exp(\eta\eta_0^{-1}-1+1/(4\eta_0^2))\,\mathrm{erfc}(\eta-\eta_0+1/(2\eta_0))\\ +(\eta-\eta_0^2/\eta)\,\mathrm{erfc}\,\eta+(\eta_0/\eta)(i\,\mathrm{erfc}(\eta-\eta_0)\\ -\eta_0^{-1}i^2\,\mathrm{erfc}(\eta-\eta_0)+1/4\eta_0)]\sin^2\phi \quad (5)\end{aligned}$$

where $\eta = R/2\sqrt{t}$ and $\eta_0 = R_0/2\sqrt{t}$.

Furthermore, Ganapathy showed in Fig. 4(b) a steady streamline pattern for $Ra = 6$, where the streamlines are multi-cellular and concluded that the second cell appears for $Ra > 3$. We should remember, however, that the perturbation analysis cannot give results which differ significantly from those given by the basic solutions (namely, ψ_0 and t_0). The streamline pattern shown in Fig. 4(b) is too different from that given by the basic solution and it is clear that the expansion (1) diverges at least for $Ra > 3$ and the present perturbation analysis cannot give any aspect of the flow in this range of the Rayleigh number.

Finally, we shall confirm that the present analysis for the sphere of finite radius gives point-heat-source solutions [3, 4] when R_0 tends to zero. For this purpose, we must somewhat modify the formulation made in [1] or [2]. In fact, if we make R_0 tend to zero in equation (5), then ψ_0 tends to zero. This is due to the fact that, for $R_0 \to 0$, the total heat transfered from the sphere, $Q_T = 4\pi a^2 Q$, tends to zero. This suggests that, in order to obtain the point-heat-source results for $R_0 \to 0$, we must formulate the sphere problem using Q_T instead of Q. When $R_0 \to 0$, Q_T corresponds to the strength of heat source. Thus, we introduce the following quantities as Bejan [3] did

$$\Theta = (T'-T_\infty)kK^{1/2}/Q_T = T/(4\pi R_0^2),$$

$$Ra^{**} = \beta g Q_T K/(\alpha v k) = 4\pi R_0^2 Ra = 4\pi Ra^*$$

and rewrite (1) and (2) in the form

$$\Theta = \Theta_0(R,\phi,t;R_0)+Ra^{**}\Theta_1(R,\phi,t;R_0)+ \quad (6)$$

$$\psi/Ra^{**} = \Psi_0(R,\phi,t;R_0)+Ra^{**}\Psi_1(R,\phi,t;R_0)+ \quad (7)$$

where $\Theta_0 = T_0/(4\pi R_0^2)$, $\Theta_1 = T_1/(16\pi^2 R_0^4)$, $\Psi_0 = \psi_0/(4\pi R_0^2)$, $\Psi_1 = \psi_1/(16\pi^2 R_0^4)$ etc. We can easily verify that, as $R_0 \to 0$, Θ_0, Θ_1, Ψ_0, Ψ_1, ... agree with the results for a point heat source [3, 4].

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